MODEL GL0.2: AN IMPROVED CLEAR-SKY SOLAR IRRADIANCE ALGORITHM FOR LOWER AND UPPER ALTITUDE SITES

Part I: UV-VIS-NIR (UVNIR) MODEL

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Abstract

It is presented an upgrade of GLo model (hereafter GL0.1) developed at 2000, which assess daily cycle of clear-sky solar irradiance and is included in CPTEC satellite-based GL models for solar irradiance. GL0.1 model considers solar irradiance as the sum of a component in UVVIS spectral interval (0.4-0.7 μm) subject to absorption/dispersion by the N2+O2+O3 mixture, plus a component in solar IR interval (0.7-2.8 μm) with pure absorption by H_2O+CO_2. It must be noted that GL0.1 algorithm is suitable for sites at 1000 hPa level. Model GL0.2 has similar spectral partition enlarges surface pressure interval and improves associated algorithms.

This Part I describes improvements for UVVIS interval redefined as UVNIR λ:(0.2-0.8 μm). Detailed spectral analysis using STOCH2F stochastic model allows parameterization of integrated UVNIR irradiance for a given site, as a function of solar zenith angle and altitude.

Resumo

Apresenta-se um aperfeiçoamento do modelo GLo (no que segue, GL0.1) desenvolvido em 2000, o qual estima ciclo diurno de irradiância solar para céu claro está incluído nos modelos GL satelitais do CPTEC para irradiância solar. O modelo GL0.1 considera irradiância solar como a soma de uma componente UVVIS no intervalo espectral (0,4-0,7 μm) sujeita a absorção/dispersão numa mistura N2+O2+O3, mais uma componente IR solar em (0,7-2,8 μm) com absorção pura por H2O+CO2. O algoritmo do GL0.1 é apropriado para locais no nível de 1000 hPa. O modelo GL0.2 tem partição espectral semelhante mas amplia o intervalo de pressões de superfície e aprimora os algoritmos associados.

Esta Parte I descreve os aprimoramentos para o intervalo UVVIS, redefinido como UVNIR λ:(0,2-0,8 μm). Uma análise espectral detalhada usando o modelo estocástico STOCH2F permite parametrizar a irradiância integrada em UVNIR para um dado local, como função do ângulo zenital solar e a altitude.

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INTRODUCTION

Clear-sky solar radiation models are useful because they provide reference values for assessing performance of radiometers, for basic estimations of radiative balance, and for models including the complex effect of gases, aerosol and cloud. A number of empirical and physical models have been published (Iqbal 1973, Barbaro et al. 1988, Antonanzas-Torres et al. 2019). In general, they present parameterizations which must be corrected in order to fit local characteristics. In this context, it is desirable to have a model requiring a set of few main physical variables and parameterizations as universal as possible.

Ceballos (2000) presented a simple physical model (hereafter called GL0.1) currently included in GL 1.2 solar radiation model (Ceballos et al. 2004) for clear-sky occurrence. It considers two spectral intervals exhibiting well defined phenomena: (a) ultraviolet-visible interval UVVIS:(0.3-0.7 μm), with ozone absorption and Rayleigh scattering; (b) solar infrared IRS:(0.7-2.8 μm), with pure absorption of direct beam by H_2O and CO_2. Null aerosol load is assumed. If direct beam on the top of atmosphere (TOA) is So with zenithal angle Zo ($\mu o = \cos Zo$), $Rp(\mu o)$ is planetary reflectance for a black ground (Rs=0) in UVVIS interval and ΔS the absorption of direct beam by water vapor and carbon dioxide, then irradiance G at ground level should be

$$Go_{UVVIS}(\mu o) = \mu o \, So_{UVVIS} \, (1 - Rp - A); \quad (1)$$

$$G(\mu o) = Go_{UVVIS} / (1 - Rs \, R^*) + \mu o \, (So_{IRS} - \Delta S). \quad (2)$$

In this model, So_{UVVIS} and So_{IRS} are partitions of solar constant. Rp for UVVIS interval is deducted from a whole-spectrum Rp algorithm of Lacis & Hansen (1974); it is assumed atmospheric absorptance A=0. ΔS depends on effective water vapor path $w2= w/\mu o$ (w= precipitable water); parameterizations are deduced from Howard et al. (1956) and Roach (1961). GL0.1 algorithm is coherent with 1000 hPa pressure level; therefore, it

A three-part G-STAR document will present a revised version of GL0.1 model, labelled GL0.2. This report (Part I of the document) is concerned to 0.2-0.8 μm spectral interval. Main aspects:

- It still considers null aerosol load.
- Spectral intervals of GL0.2 are UVNIR:(0.2-0.8 μm), and IRS:(0.8-2.8 μm).
- Within UVNIR the stochastic two-flux code STOCH2F (Ceballos 2021) is used for detailed spectral description of radiative properties, including revision of counter-reflectance R* and expected irradiance at altitudes z:(0-4 km), characteristic of mountainous areas in South America.
- Go_{UVNIR} model includes planetary reflectance Rp and stratospheric absorption A.

Part II will criticize the water vapor absorption scheme used in GL0.1 and GL1.2 models and provide an updated algorithm which includes mean atmospheric pressure parameter, vertically weighted with precipitable water.

Part III will include UVNIR and IRS irradiances in GL0.2 model, with definition of geographic distribution of fundamental parameters over South America.

The model GL0.2 will be included in version GL 1.4 of GSTAR-INPE solar radiation model.

1. STOCHASTIC TWO-FLUX RADIATION TRANSFER SCHEME FOR UV-VIS-NIR (UVNIR) INTERVAL

Consider the atmosphere divided in N (horizontal) homogeneous layers, with direct beam So (solar constant) impinging on the top of atmosphere (TOA) with zenith angle Zo (cosZo = μo). Ground exhibits spectral reflectance Rs(λ). For a site at altitude *z*, atmospheric depth in the geometric interval (z, ∞) corresponds to an optical depth τ for radiation with wavelength λ. Given monochromatic direct beam So_λ at TOA ([1]), Beer's law describes its attenuation as

$$S_\lambda(\tau) = So_\lambda \exp(-\tau/\mu o). \quad (1.1)$$

Radiative transfer produces not only attenuation of direct beam, but also generates diffuse flux. Considering monochromatic radiative transfer (for the sake of simplicity: not using the subscript λ), the n-th layer located between optical depths $\tau=\tau_n$ and $\tau=\tau_n+\Delta_n$ will have *diffuse irradiances* $E\uparrow$ (ascendant) and $E\downarrow$ (descendant) described by

$$dE\downarrow/d\tau' = -a_{11}\, E\downarrow + a_{12}\ E\uparrow + a_{13}\, S(\tau_n+\tau'), \quad (1.2.a)$$

$$dE\uparrow/d\tau' = -a_{12}\, E\downarrow + a_{22}\ E\uparrow + a_{23}\, S(\tau_n+\tau'), \quad (1.2.b)$$

$$S(\tau_n+\tau') = So \exp(-\tau/\mu o) = S(\tau_n) \exp(-\tau'/\mu o). \quad (1.2.c)$$

Equations (1.2) are derived from the general Radiative Transfer Equation RTE (Liou 2002, Zdunkowski et al. 2007) and constitute the two-flux model describing diffuse irradiances at optical depths τ':[0 Δ_n]. Coefficients a_{ij} are in fact dependent on τ', but can be assumed as a set of constant values within each layer, depending on simplifying hypotheses (Ceballos 1988, Meador and Weaver 1980, Liou 2002). The complete solution of eqs. (1.2) within N layers for $E_n\downarrow$, $E_n\uparrow$ must fulfill boundary conditions

$$E_1\downarrow(0)=0 \quad \text{(first layer)} \quad (1.2.d)$$

$$E_n\downarrow(\Delta_n) = E_{n+1}\downarrow(0) \quad \text{(top of n-th layer, n>1)} \quad (1.2.e)$$

$$E_{n+1}\uparrow(0) = E_n\uparrow(\Delta_n) \quad \text{(base of n-th layer, n<N)} \quad (1.2.f)$$

$$E_N\uparrow(\Delta_N) = Rs\,[\mu o\, S(\tau_N) + E_N\downarrow(\Delta_N)]. \quad \text{(base of N-th layer).} \quad (1.2.g)$$

This procedure allows retrieval of irradiance profile but loses information about internal interactions among layers and radiative transfer. These interaction can be described by the stochastic scheme described in the next section.

1.1. Foundations of a stochastic two-flux (STOCH2F) radiative transfer model

Basic concepts of a stochastic two-flux model in N-layered atmospheres and some applications were were developed in Ceballos (1986, 1989) and Souza et al. (19xx). Ceballos (2021)

[1] S_λ is a spectral density measured in units W m^{-2} μm^{-1} *along the beam* (normal-incidence irradiance or directional flux), while direct irradiance or "direct radiation" is $E_\lambda = \mu o\, S_\lambda$.

presented a detailed description of STOCH2F code applied to radiative properties of clean atmospheres in 0.2-0.7 μm wavelength interval.

Considering irradiances $E\downarrow(\tau')$, $E\uparrow(\tau')$ *referred to direct irradiance on the top of a layer*, that is $\mu o\, S(\tau_n)$, they become variables $\zeta\downarrow(\tau')$, $\zeta\uparrow(\tau')$ and eqs. (1.2.a-c) can be written

$$d\zeta\downarrow/d\tau' = -a_{11}\,\zeta\downarrow + a_{12}\,\zeta\uparrow + a_{13}\exp(-\tau'/\mu o), \qquad (1.3.a)$$

$$d\zeta\uparrow/d\tau' = -a_{12}\,\zeta\downarrow + a_{22}\,\zeta\uparrow + a_{23}\exp(-\tau'/\mu o). \qquad (1.3.b)$$

For a single isolated layer, boundary conditions are $\zeta\downarrow(0) = 0$, $\zeta\uparrow(\Delta_n) = 0$. Their solutions yield transfer properties of the layer *referred to incidence of direct radiation*: To (diffuse transmittance), Ro (layer reflectance), Ao (layer absorptance):

$$\zeta\downarrow(\Delta_n) = To_n, \quad \zeta\uparrow(0) = Ro_n, \quad \exp(-\Delta_n/\mu o) + To_n + Ro_n + Ao_n = 1. \qquad (1.4)$$

Figure 1 illustrates how a beam of direct photons incident on the top of forth layer generates these four classes of flux.

For diffuse flux incident on top of a layer, $a_{13} = a_{23} = 0$ in eqs. (1.3). No direct beam is included; a single isolated layer requires the boundary conditions $\zeta\downarrow(0) = 1$, $\zeta\uparrow(\Delta_n) = 0$. Os fluxos through boundaries are diffuse transmittance T and reflectance R together with absorptance A, yielded by ([2])

$$\zeta\downarrow(\Delta_n) = T_n, \quad \zeta\uparrow(0) = R_n, \quad T_n + R_n + A_n = 1. \qquad (1.5)$$

Consider the atmosphere as an N-layered system. Incidence of a beam of photons on top of atmosphere (TOA) leads to a set of "initial diffuse photon fractions", with N descending diffuse fluxes at the bottom of layers, N ascending diffuse fluxes (at bottom of the layers also) and N absorbed fluxes (inside the N layers), plus a fraction escaping into sky from the first layer. This means a total of 3N+1 "initial diffuse states" with two noteworthy observations: at the base of N-th layer there exist 1) null ascending flux, and 2) outgoing flux $E_{exit} = \mu o\, So \exp(-\tau/\mu o)$ is a remaining direct irradiance.

Now let us include ground as a part of the whole system. Considering absorption within ground as "diffuse state", then 1) a fraction Rs of E_{exit} is located as upward diffuse flux at N-th layer base, 2) a fraction (1-Rs) is located at ground. In this way, direct irradiance at TOA is entirely transferred to 3N+2 diffuse states. We can consider that two-flux radiative transfer exhibits a (3N+2)-dimensional initial vector $\mathbf{P}_o$.

The initial diffuse photons start an up-and-down random walk until stopping in one of N+2 absorbing states. Single transfer process is realized jumping to neighboring layer; the physical properties reflectance, transmittance and absorptance (R_n, T_n, A_n) of the n-th visited layer can be assessed from eqs. (1.3) with the same boundary conditions but $a_{13} = a_{23} = 0$ (no direct radiation)([3]). Therefore, successive steps in random walk describe a sequence of vectors $\mathbf{P}_o$, $\mathbf{P}_1$, $\mathbf{P}_2$, … following a first-order Markov process with transition probabilities described by basic properties (R, T, A).

In lieu of speaking about evolution of fractional numbers of photons, let's think about the process in terms of probability of states of a single photon. Solutions $\zeta_n\downarrow$, $\zeta_n\uparrow$of equations (1.3) are *conditional probabilities* for a direct photon being located as a diffuse photon at the base (descending), at the top (ascending) and inside a layer, *given the incidence on the top of that*

[2] Provided that incident flux be isotropic, properties T, R, A are the same for incidence on the top (descendant flux) and on the bottom (ascendant flux).

[3] Note that (R,T,A) are *not* (Ro,To,Ao), and do not depend of incident angle Zo.

layer. Probability of incidence at the top of that layer is $\exp(-\tau_{n-1}/\mu o)$; thence initial probabilities composing (3N+2)-dimensional vector $\mathbf{P}o$ are

Prob{absorbed, sky}= Ro1	{state 1}	(1.5a)
Prob{↓, n-th layer} = $To_n \exp(-\tau_{n-1}/\mu o)$;	{states 21-38 in Figure 1}	(1.5b)
Prob{↑, n-th layer} = $Ro_n \exp(-\tau_{n-1}/\mu o)$;	{states 40-56}	(1.5c)
Prob{absorbed, n-th layer}= $Ao_n \exp(-\tau_{n-1}/\mu o)$;	{states 3-20}	(1.5d)
Prob{↑, ground)= Rs $\exp(-\tau_N/\mu o)$;	{state 39}	(1.5e)
Prob{absorbed, ground}= (1-Rs) $\exp(-\tau_N/\mu o)$;	{state 2}	(1.5f)

Figure 1 (right) illustrates the case of a 18-layer atmosphere, properly numbering the 56 diffuse states.

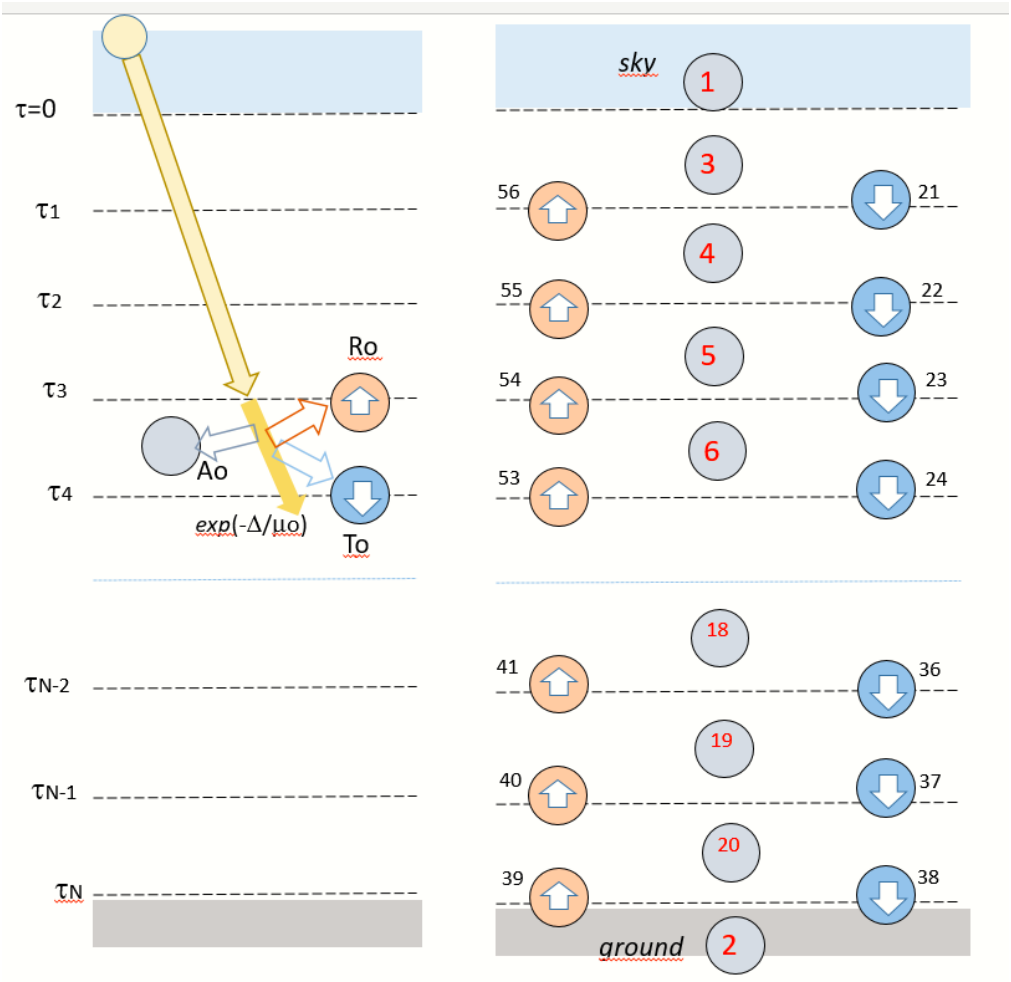

Figure 1. Two-flux radiative transfer. (left) Generation of initial diffuse photons. (right) States set for transitions a first-order Markov process, in an 18-layer atmosphere.

Figure 1 helps to describe the corresponding Markov transition matrix **Q**(54×54). Diffuse photons transit to neighboring states such that, for instance, Q(22,n)= 0 except for Q(22,5)= A_3; Q(22,23)= T_3; Q(22,55)= R_3. Absorption states n= 1-20 have Q(n,n)=1 that is, no further transitions are allowed. After infinite transitions photons are located at states 1-20 only.

Evolution $\mathbf{P}_o$, $\mathbf{P}_1$, …, $\mathbf{P}_{n\to\infty} = \pi$ is described by

$$\mathbf{P}_n = \mathbf{P}_o\,\mathbf{Q}^n, \qquad \pi = \mathbf{P}_o\,\mathbf{Q}^\infty \tag{1.6}$$

Photons fate is unavoidable absorption at one of N+2 specific states, for which n-th components of vector π are not null. Choosing n=1 for "sky state" and n=2 for "ground state", and given the spectral irradiance $\mu o\, S\lambda$ at TOA within interval $\Delta\lambda$, then fundamental values for irradiances are

$$Eo = \int_{\Delta\lambda} \mu o\, S\lambda\, d\lambda \rightarrow \quad \text{incident irradiance at TOA} \tag{1.7}$$

$$Er = \int_{\Delta\lambda} \pi_1(\lambda)\, \mu o\, S\lambda\, d\lambda \rightarrow \quad \text{reflected irradiance at TOA} \tag{1.8}$$

$$Eg = \int_{\Delta\lambda} \pi_2(\lambda)\, \mu o\, S\lambda\, d\lambda \rightarrow \quad \text{irradiance absorbed at ground} \tag{1.9}$$

$$G = \int_{\Delta\lambda} (1 - Rs)^{-1}\, \pi_2(\lambda)\, \mu o\, S\lambda\, d\lambda \rightarrow \quad \text{global radiation incident at surface} \tag{1.10}$$

Numbering with 3 to N the absorption states in layers 1 to N, if the base of Ns-th layer is chosen as boundary between stratosphere and troposphere, then absorption probability and absorbed irradiance at stratosphere and troposphere are

$$\pi_{strat} = \sum_{n=3,Ns} \pi_n(\lambda) \rightarrow \qquad Estrat = \int_{\Delta\lambda} \pi_{strat}(\lambda)\, \mu o\, S\lambda\, d\lambda\,, \tag{1.11}$$

$$\pi_{trop} = \sum_{n=Ns+1,N} \pi_n(\lambda) \rightarrow \qquad Etrop = \int_{\Delta\lambda} \pi_{trop}(\lambda)\, \mu o\, S\lambda\, d\lambda. \tag{1.12}$$

Quotient between iradiances (1.8) to (1.12) and irradiance at TOA (1.7) yields corresponding fractional fluxes absorbed in those atmospheric regions.

1.2. Atmospheric profiles and optical parameters

Figure 2 illustrates profiles of air density and O_3 concentration for tropical (TROP) and mid-latitude winter (MLW) atmospheres, according to classical reference of McClatchey et al. (1972), divided in eighteen layers. Density profiles are similar, but O_3 profile shows differences, even when considering versions of McClatchey or Lacis and Hansen (1974), this last labelled "L&H".

For two-flux radiative transfer within a layer the fundamental parameters are the triad (τ, ω, g): optical depth, single-scattering albedo and asymmetry factor, allowing definition of coefficients a_{ij} in Eqs. (1.3) in order to assess transfer properties of a given layer (see Annex 2).

Optical depth for a layer with geometrical thickness Δz is given by addition of Rayleigh (scattering) and O3 (pure absorption) effects

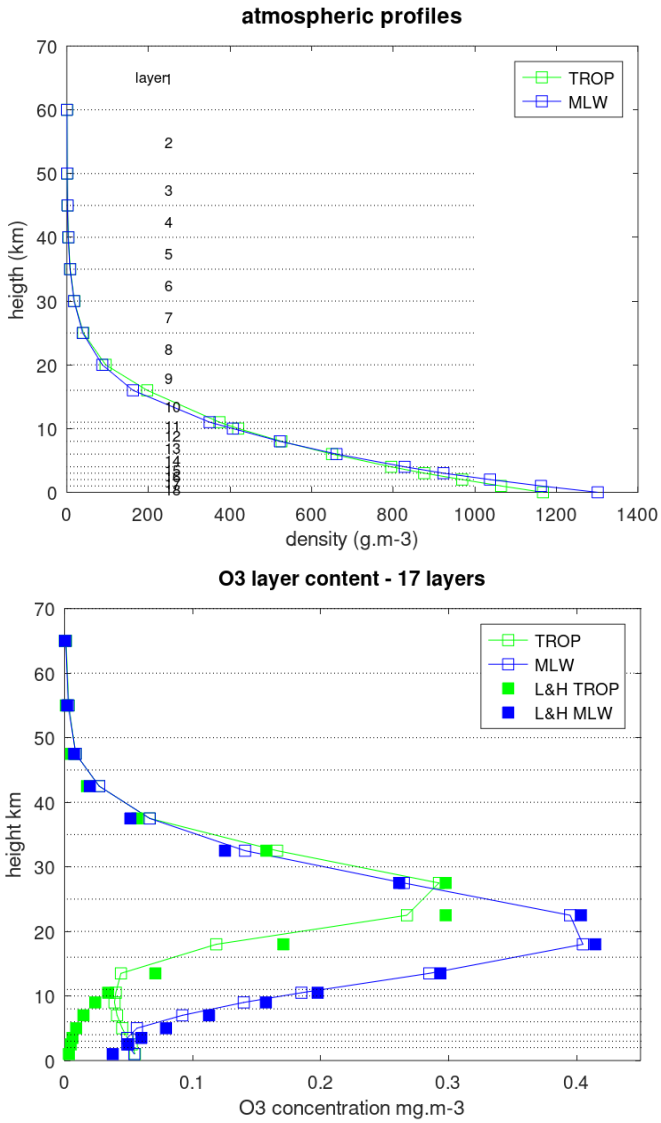

Figure 2. (*above*) Density profile for Tropical and Mid-latitude winter atmospheres at surface and at the top of 18 layers. (*below*) Similar for O3 density.

$$\tau = \tau_R + \tau_3 \,, \qquad (1.13a)$$

$$\tau_R = 0.00867\ \lambda^{(-4.15+0.2\,\lambda)}\ (\Delta P/P_o), \qquad \lambda \text{ in } \mu m, \qquad (1.13b)$$

$$\tau_3 = N_3\ \sigma_3\ \Delta z, \quad [N_3] = \text{molec m}^{-3}, \ [\sigma_3] = \text{m}^2 / \text{molec}. \qquad (1.13c)$$

Eq. (1.13b) improves usual algorithm for Rayleigh scattering optical depth (Paltridge e Platt 1976; Bodhaine et al. 1999); ΔP refers to pressure thickness of the layer, and Po= 1013 hPa. In Eq. (1.13c), N_3 can be deduced from McClatchey profiles (Annex 2) and molecular cross section σ_3 can be found in Ceballos (2021). Parameter ω describes probability of scattering in a single dispersion, estimated by

$$\omega = \tau_R \ / (\tau_R + \tau_3). \qquad (1.14)$$

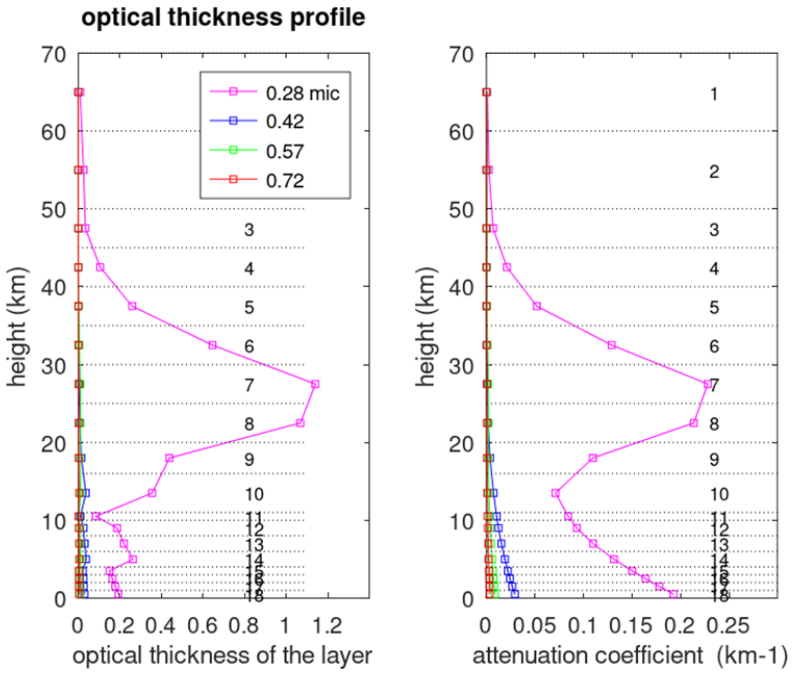

Figure 3. Optical depth of 18-layer TROP atmosphere, for 4 wavelengths, and attenuation coefficient for each layer.

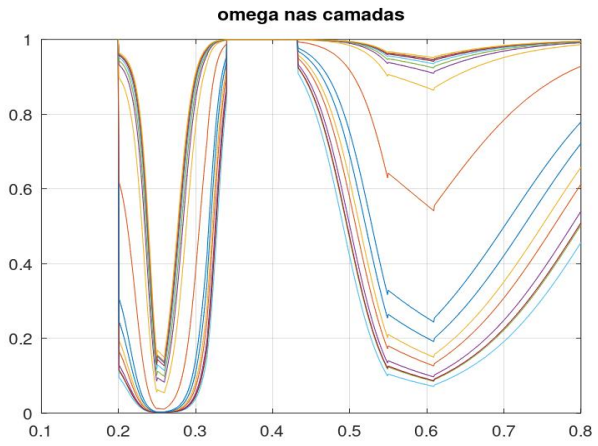

Figure 4. Spectrum of single-scattering coefficient within 0.2-0.8 μm interval for the 18 layers (1 to 18 from lower to higher ω values).

Figure 3 shows profile of τ and ω for the TROP McClatchey atmosphere. Optical depth seems to exhibit fluctuations at 6 and 14 km; actually, this is due to different geometric thicknesses δz; when represented linear attenuation coefficient k = τ/δz we note the smooth variation of optical profile within the atmosphere. It must also be noted the influence of O_3 absorption for air+O_3 mixture in the stratosphere, mainly in UV wavelengths as 0.28 μm, while air (O_2+N_2) Rayleigh scattering is predominant in the troposphere.

The ω spectrum in UV-VIS-NIR interval 0.2-0.8 μm shows the influence of the strong Hartley band in 0.20-0.315 μm, followed by the weaker Huggins band in 0.315-0.35 μm. Solar radiation with λ:(0.2-0.32 μm) must be strongly absorbed in every layer, *notably the incoming radiation* in the upper atmosphere. Above 0.36 μm radiative transfer is conservative except in

the Chappuis band interval (0.43-0.78 μm), which is weakly absorptive in the lower troposphere but can be proportionally very active in the stratosphere (although O_3 shows low optical depth, there is much lower conservative interaction with O_2+N_2 molecules).

Hereafter, we consider z= 16 km as the limit between stratosphere and troposphere (base of ninth layer).

1.3. Building the stochastic model

McClatchey profiles were used in order to define 18 atmospheric layers (see Annex 1). Figures 2 illustrate pressure, air density and O3 profiles. Each layer defines a triad {τ ω g} where τ is optical depth, ω is single-scattering albedo and g is asymmetry factor. These ones are fundamental parameters for defining a_{ij} coefficients in Eqs. (1.3), as shown in Annex 2.

The scheme of stochastic states is illustrated by Figure 1. Solutions of Eqs. (1.3) for each layer (with upper and bottom blackbody boundaries) define direct transmittances and properties To, Ro, Ao (To, Ro being initial diffuse photon fluxes) allowing initial allocation of photons following Eqs. (1.5) for definition of initial vector Po with dimension N = 3×18 + 2= 56. Absorbing states stay in sky (1), ground (2) and 18 layers (3-20). Solutions of Eqs. (1.3) with the same parameters a_{ij} but null direct incidence provide transition probabilities applied to k-th layer define properties T_k, R_k, A_k for diffuse photons; they fill the transition matrix **Q**(N×N). For instance, fourth layer has transition probabilities Q(k1, k2) = Q(k1→ k2)

Q(23→24)= T_4, Q(23→ 54)= R_4, Q(23→ 6)= A_4,

Q(53→ 54)= T_4, Q(53→ 24)= R_4, Q(53→ 6) = A_4.

Absorbing states k1= 1-20 have Q(k1→k1))= 1. Other transitions have null probability.

The final vector was obtaines for n= 80 iterations of transition process, that is

$$\pi = \mathbf{P}o\ \mathbf{Q}^n, \qquad n= 80. \tag{1.15}$$

The component π_2 describes groud absorption, while G = π_2/(1-Rs) describes global irradiance, which includes the effects of first impact on ground (Go) and of multiple reflections ground-atmosphere. The first is provided by π_2 if Rs=0. For the second, a fundamental parameter is atmospheric counter-reflectance; this provided by π_2 considering Rs=0 and components Po(k) of initial vector being Po(k=39)= 1, Po(k≠39)=0, that is, first state emerging from ground.

Abovementioned procedure implies in having ground at z=0 (or po= 1013 hPa). If ground is located at a different altitude (say, the bottom of ko-th layer, ko<18), properties of global irradiance are obtained in the same manner, except that layers k>ko must be eliminated. For instance, a scheme with atmospheric bottom at z= 2 km (see profile in Annex 1) implies in 16 layers with po= 805 hPa, N= 3×16+2 = 50 states, absorbing states 1-18, downward states 19-34, upwelling states 35-50. Radiative properties of each layer are the same abovementioned for layers 1-16. Clearly, Q(34,2)= 1-Rs, Q(34,35)= Rs.

1.4. Results

Fluxes for black ground (Rs= 0).

Figure 5 illustrates spectral probabilities and irradiances for black ground and surface pressure po= 1013 hPa. The first observation is that UV absorption virtually reduces UV solar constant from 111 to effective 95 Wm^{-2} in λ:(0.3-0.4 μm), and UVNIR from 772 Wm^{-2} in λ<0.8 μm interval to 757 Wm^{-2} over an effective interval λ:(0.3-0.8 μm) (solar spectrum following Gueymard 2004).

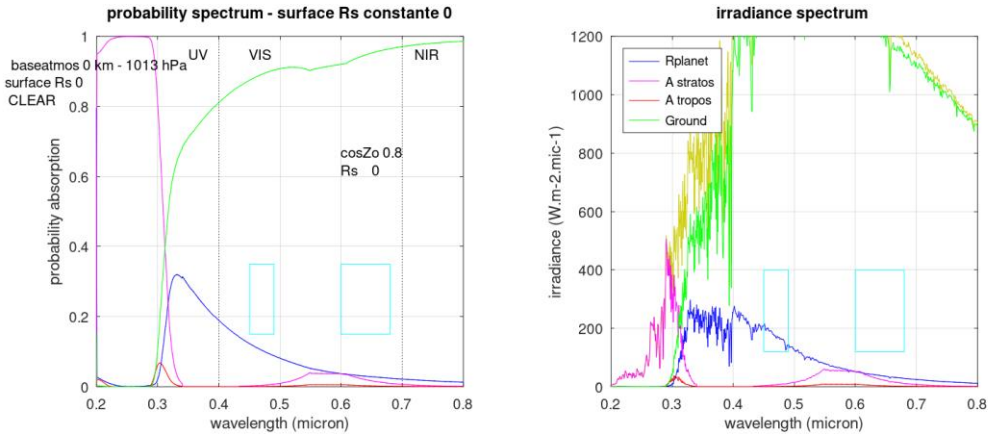

Figure 5. (*left*) Spectrum of absorption probabilities (at sky, within stratosphere, within troposphere and at ground, ground reflectance being Rs=0. (*right*) Corresponding spectral irradiances.

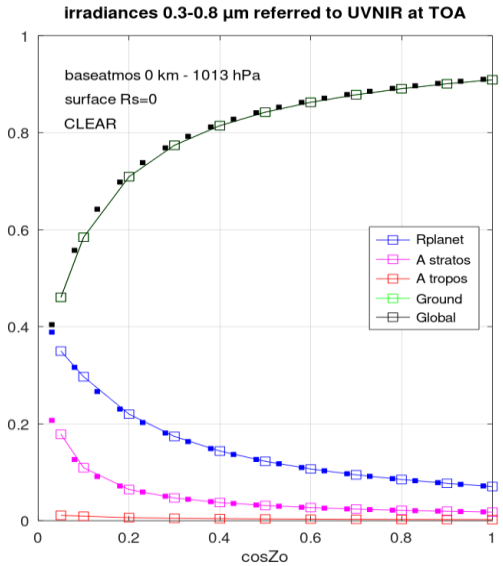

Figure 6. Dependence of (integrated) relative UVNIR irradiances and absorptions on solar zenithal angle. Tropical atmosphere, ground level (po= 1013 hPa) with black ground.

Figure 6 shows the results for a tropical atmosphere with pressure 1013 hPa at z=0, integrated over λ:(0.3-0.8 μm). Given that absorption within troposphere is negligible, energy balance in UVNIR interval becomes

$$\mu o\ So_{UVNIR} \approx (Rp + As)\ \mu o\ So_{UVNIR} + G_{UVNIR}. \quad (1.15)$$

Rp and As are planetary reflectance and stratospheric absorptance, referred to UVNIR irradiance on TOA. Defining atmospheric (global) transmitance as Kto, we have

$$Kto = G_{UVNIR}/\mu o\ So_{UVNIR} = 1 - Rp - As. \quad (1.16)$$

Figure 6 includes numerical results of STOCH2F (empty squares), as well as the interpolations of Rp and As (filled squares) found by fitting functions

$$y = (a + b\ \mu o + c\ \mu o^2)^{-1}. \quad (1.17)$$

This expression seems excellent for model parametrizations.

Rp, As, Go values for various atmospheric columns is obtained by successive elimination of lowest pressure level in profile data (see Annex 1), up to z= 6 km altitude. Resultant bases of atmosphere (with 18 to 13 layers) follow the Table below.

```
Yz= [...
%   z  dens   po   O3   dens   po   O3
    06  650  492 4.30    661  463  6.4; ...
    04  795  633 4.70    828  608  4.9; ...
    03  876  715 5.10    923  694  4.9; ...
    02  969  805 5.40   1037  790  4.9; ...
    01 1064  904 5.60   1162  897  5.4; ...
    00 1167 1013 5.60   1301 1018  6.0];
```

STOCH2F applied to these atmospheres yields the sequence of coefficients (ar, br, cr) for reflectance and (aa, ba, ca) for absorptance, as follows.

```
abcRA= [...
% n z   po       ar      br     cr          aa    ba     ca
  0 0 1013    2.2186  11.777  .1778      2.953 62.92 -10.32; ...
  1 1  904    2.2580  13.222  .1679      2.939 63.28  -9.97; ...
  2 2  805    2.2987  14.874  .1550      2.926 63.63  -9.62; ...
  3 3  715    2.3409  16.775  .1368      2.912 63.97  -9.27; ...
  4 4  633    2.4324  19.345  .0911      2.899 64.31  -8.92; ...
  5 6  492    2.4762  24.484  .0706      2.869 65.02  -8.32];
```

In the first column, n means the number of levels eliminated and (z, po) the altitude and pressure of the new base of atmosphere. Eq. (1.17) can be expressed as

$$Rp = Ar / (1 + Br\ \mu o + Cr\ \mu o^2); \quad (1.18a)$$

$$As = Aa / (1 + Ba\ \mu o + Ca\ \mu o^2). \quad (1.18b)$$

We can expected that coefficients (A, B, C) vary with total pressure (that is, with the amount of optical path in the column); Figure 7 illustrates dependence on po.

Concerning Rpo, 1) Ar can be considered nearly constant within 5%; 2) Br is the main factor in Eq. (1.18a) and clearly depends on pressure po; 3) Cr is also variable with po, but as a weaker

contribution to Rp. In order to assess Aso in stratosphere (eq. 1.18b), (Aa, Ba, Ca) can be considered constant within 1-2%. Interpolated coefficients (pressure po in hPa) are:

P= po/1000; [po] = hPa;

Ar= 0.353 + 0.099 P, or 0.43 ± 0.02; (1.19a)

Br= $\beta\ P^{\alpha}$, β= 5.369, α= -0.860;

Cr= 1.309 P^3 – 3.530 P^2 + 3.216 P – 0.915;

Aa= 0.342± 0.003; (1.19b)

Ba= 21.7 ± 0.4,

Ca= -3.28 ± 0.20.

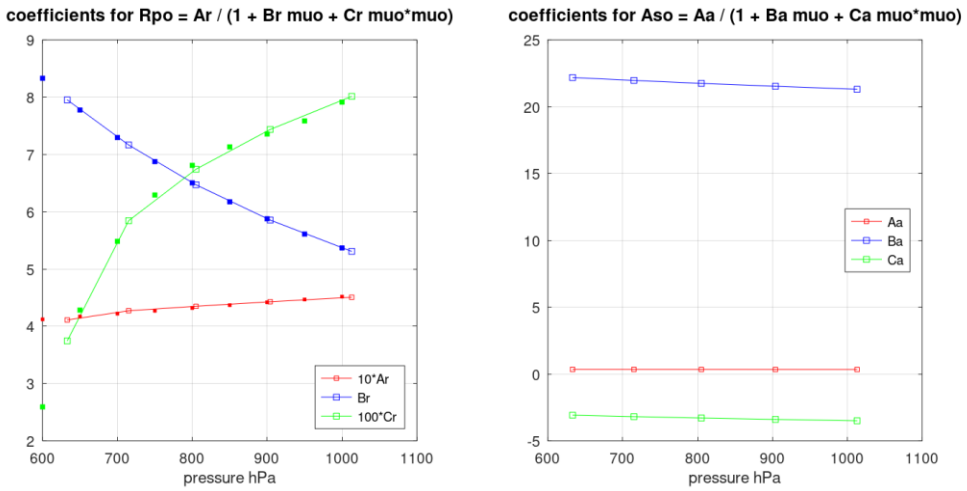

Figure 7. Variability of coefficients (A, B, C) with total pressure at ground level. Empty squares indicate values obtained by STOCH2F. Filled squares illustrate interpolations (Eqs. 1.19).

Atmospheric counter-reflectance R*

Atmospheric base levels zbase= 0, 1, 2, 3, 4 km were considered, thus the number of layers was N= 18, 17, 16, 15, 14. Initial state was 2N+3 (zbase, ascending). Black ground (Rs=0) implies in escape to sky (transmittance T*) and return probability (counter reflectance R*) given by π_1 and π_2 components. Absorption in the atmosphere is assessed by A*= 1- π_1 - π_2. Figures 8 show the spectral results for zbase at 0 km and 4 km. Following Figures 5 and 8 we can assume an effective spectral lower limit λo= 0.3 μm; below this limit, not only O3 absorption is high but irradiance arriving to ground is negligible.

It is noteworthy that $R_{troplayer}$ = R* estimated for a Rayleigh atmosphere in a troposphere with top level at 50 hPa (Eq. A2.7b) closely follows estimates of STOCH2F for $\lambda > \lambda o$. This should be expected, since upward diffuse photons going into stratosphere exhibit a very low probability of return to the surface (Figure 8 includes the curve $R^*_{stratos} = \pi_2$ obtained for initial state upward at the base of stratosphere, be 39 if N= 18, zbase=0, or 31 if N=14, zbase= 4 km).

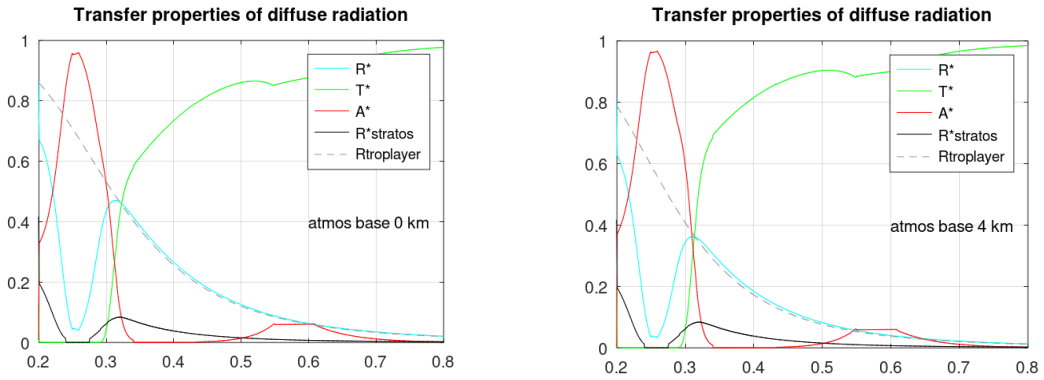

Figure 8. Transmitance T*, Counter-reflectance R* and absorption A* for a diffuse photon starting upward at ground level. Two TROP atmospheres extend through z:(zbase - ∞), with zbase= 0 and zbase= 4 km. Also included: (a) probability of return to ground, starting upward at the base of stratosphere (black line); (b) counter-refectance Ro* for a Rayleigh atmosphere with the top at 50 hPa be z:(zbase - about 20 km), dashed gray line.

Given absorption Go= π_2(Rs=0) in a black ground, no counter-reflection happens; thence for R_S>0 we can assume Go as a first contribution to surface irradiance followed by multiple atmospheric reflections. The monochromatic version of an UVNIR model for global radiation becomes

$$G = Go + Go.\ RsR^* + (Go\ RsR^*).\ RsR^* + \ldots = Go / (1 - Rs\ R^*). \qquad (1.20)$$

This is equivalent to STOCH2F estimate π_2(Rs>0)/(1 – Rs). Go is a function of (λ, μo), R* being only of λ. We can define a mean value R** over UVNIR interval λ:(0.3-0.8 μm); in terms of irradiances Kto, Kt,

$$Eoo(\mu o) = \int_{UVNIR} \mu o\ So_\lambda\ d\lambda, \qquad (1.21a)$$

$$Eo(\mu o) = \int_{UVNIR} Go(\lambda;\mu o)\ \mu o\ So_\lambda\ d\lambda = Kto\ Eoo, \qquad (1.21b)$$

$$E(\mu o) = \int_{UVNIR} Go(\lambda;\mu o)\ (1 - RsR^*)^{-1}\ \mu o\ So_\lambda\ d\lambda = Kt\ Eoo, \qquad (1.21c)$$

$$Kt = Kto / (1 - Rs\ R^{**}). \qquad (1.21d)$$

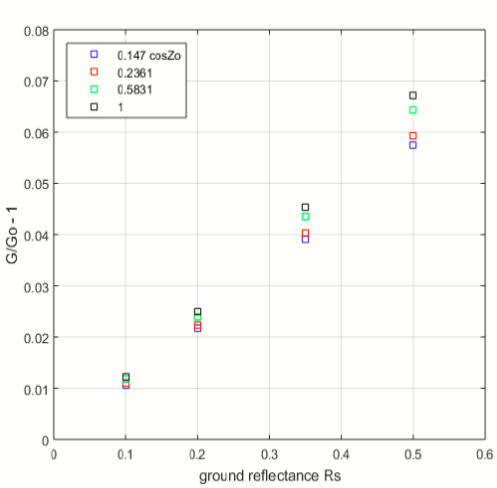

Figure 9. Evidence of linearity of (E/Eo – 1) *versus* Rs for different values of Zo (Source: Ceballos 2021)

Considering a TROP McClatchey atmosphere, surface pressure 1013 HPa, Ceballos (2021) found evidence of nearly constant values of R**. Figure 9 presents data of (Kt/Kto – 1) versus Rs for several values of μo and Rs for interval λ:(0.2-0.8 μm). It is evident a linearity of the whole set yielding constant values R** *independent of Rs* but shows some variability with Zo (R** ≈ 0.119 for Zo>76º and R** ≈ 0.134 for Zo<60º). In order to inspect variability of R** with altitude and solar zenith angle, we considered Rs= 0.3 and λ:(0.3-0.8 μm) as effective interval. Figure 3 resumes results of using spectral probabilities Go, G and Eqs. (1.21) allowed to build Figure 10.

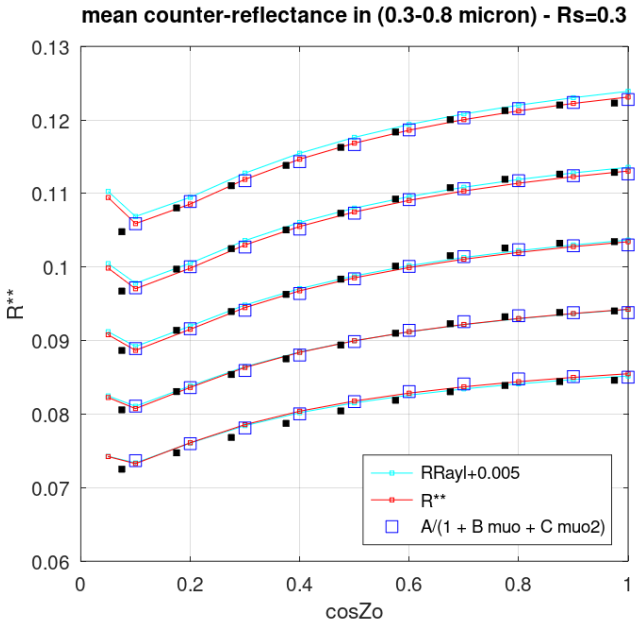

Figure 10. Mean counter-reflectance over UVNIR, following (a) R*(λ) estimate following Rayleigh scattering; (b) R*(λ) estimate from STOCH2F; (c) interpolation of case (b). Black squares show result of a general interpolation (see text).

- Mean counter-reflectance varies slowly with μo = cosZo, and with altitude z (km), between 0.073 and 0.123. Curves in Figure 10 correspond to the sequence *z*= 0, 1, 2, 3, 4 km for the base of troposphere. Irregular behavior for μo< 0.1 (Zo>84º) is due to shortcomings of STOCH2F algorithm for near horizontal solar beam.
- Rayleigh-based and STOCH2F values R* yield nearly the same mean value: R**(STOCHF)= R**(Rayleigh2F) + 0.005;
- R** can be accurately described by an algorithm R**(μo, z)= A/(1 + B μo + C μo^2), where A, B, C are functions of altitude. Nevertheless, B and C are much less variable than A, replaced by their mean value in z:(0-4 km).

An accurate final algorithm for TROP atmosphere (black squares in Figure 10) is

R**(μo, z)= A(z)/(1 + Bmed μo + Cmed μo^2), (1.22)

A= 0.102 – 0.008 z(km), Bmed= -0.334±0.020, Cmed= 0.171±0.015.

Application: a daily cycle for GLuvnir

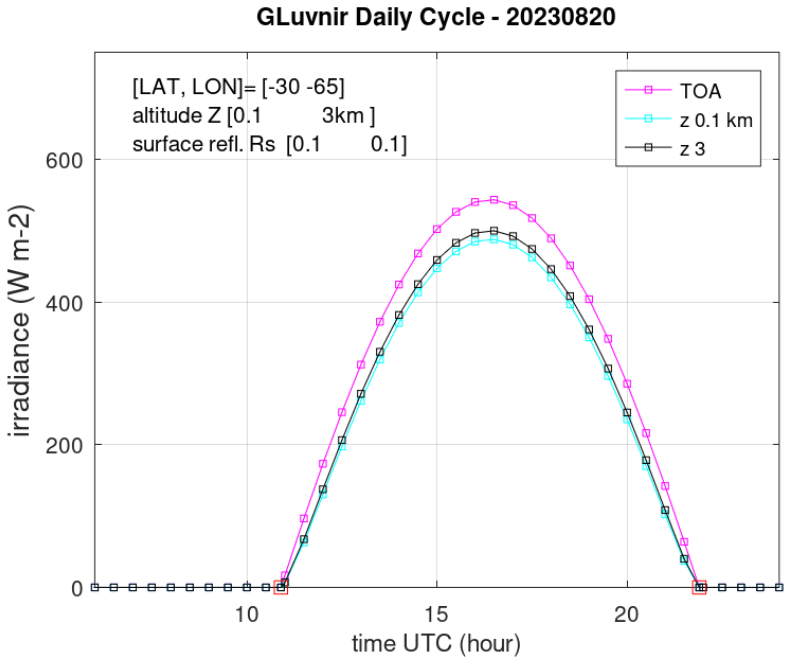

Figure 11. Daily cycle of GLuvnir for 20230820 at a site located at -30º latitude, -65º longitude, for two altitudes: 100 m and 3000 m. MLW atmospheric profile was assumed.

The set of resultant parameterizations was applied to estimate Kt(μo) (eq. 1.21d) and corresponding irradiance GLuvnir= μo.S.Kt at altitude z= 100m and 3000m. Figure 11 illustrates results.

1.5 CONCLUSIONS

Ozone absorption+Rayleigh attenuation in UV, together with weak spectral irradiance at TOA, strongly reduce surface global irradiance Go in $\lambda<0.3$ μm; therefore, $\Delta\lambda$:(0.3-0.8 μm) should be assumed as effective UVVISNIR (here called UVNIR) interval. Tropospheric pressure profile p(z) is not actually important, because optical depth τ (and scattering processes) depends only on pressure value, that is, total optical path. Therefore, Tropical (TROP) McClatchey profile was considered.

Clear-sky irradiance GL in UVNIR:(0.3-0.8 μm) for a clean atmosphere (without aerosol) can be described by simple algorithms. For solar zenith angle Zo (μo = cosZo) with black surface (Rs= 0) and altitude z km (corresponding pressure po hPa),

$$GLo(\mu o) = \mu o\, S_{UVNIR}\, (1 - Rpo - Aso),$$

$$Rpo = Ar / (1 + Br\, \mu o + Cr\, \mu o^2),$$

$$Aso = Aa / (1 + Ba\, \mu o + Ca\, \mu o^2),$$

- S_{UVNIR}= 757 W m^{-2}: directional solar flux in UVNIR interval, mean Earth-Sun distance
- μo= cosZo, Zo solar Zenith angle
- Rpo(μo, po): local planetary reflectance related to TOA irradiance in $\Delta\lambda$
- Aso(μo, po): local stratospheric absorption related to TOA irradiance in $\Delta\lambda$
- Ar, Br, Cr, Aa, Ba, Ca: coefficients depending on pressure at atmospheric base. Described by Eqs. (1.19).

For actual surfaces (Rs>0 typical value in $\Delta\lambda$), multiple reflections surface-troposphere increase original irradiance GLo through a simple algorithm

$$GL(\mu o, Rs) = GL(\mu o)/(1 - Rs\, R^{**}),$$

$$R^{**}= A(z) / (1 + Bmed\, \mu o + Cmed\, \mu o^2),$$

- Rs = ground reflectance in UVNIR interval
- R**= mean value of atmospheric counter-reflectance in UVNIR, Eqs. (1.22). It is essentially not dependent on Rs but on altitude z, and weakly on μo.
- R** is counter-reflectance R*(λ) weighted with spectral GLo(λ) obtained by STOCH2F. It is noteworthy the fact that R* obtained through STOCH2F as well as a simple algorithm based on 2-flux equations applied to a (po – 50 hPa) atmospheric layer allow spectral integration of GLo/(1 – Rs R*) with the same quality of results.

When applying GL 0.2 parameterizations to sites of different altitudes, it is seen that a clean clear-sky atmosphere has significant influence on UVNIR irradiance. For the chosen day and local, it is about 55 Wm^{-2}; however, the difference between two altitude cases (z= 100m and z= 3000m) does not exceed 12 Wm^{-2}. The results suggest that clean atmosphere has a typical influence on surface irradiance, with lower impact of altitudes between 0 and 4000m. Higher impact of altitude could be expected concerning aerosol concentrated within planetary boundary layer (z< 2000m), and/or absorption by H_2O vapor in IRS interval. Precipitable water shows a

steeper decrease with altitude. This last influence is the subject of Part II of this research for GL 0.2 model.

Routines (Octave compatible) are included in Annex, for estimation of GL_{UVNIR} as a function of date and hour, and local latitude/longitude/altitude.

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ANNEX 1: McClatchey profiles

TROP: Tropical, MLW: MidLatitude Winter

```
    % ------ 18 layers original McClatchey ---

%   z    --- TROP -----------   --- MLW --------------
%    z   ro    p   O3   U       ro    p     O3   U
Yz= [...
    70  .09  .058 .008 1.4e-7   0.07 .047  .01  1.4e-7; ...
    60  .60   .40 .21  ...      0.48 .300  .15  ... ; ...
    50  1.1  .85  .43  6.3e-6   0.89 .682  .43  6.3e-6; ...
    45  2.1  1.59 1.3  1.9e-5   1.74 1.29  1.3  1.9e-5; ...
    40  4.2  3.0 04.1  4.3e-5   3.62 2.53  4.1  4.3e-5; ...
    35 08.6  6.0 09.2  1.1e-4   7.92 5.18  9.2  1.1e-4; ...
    30 18.3 12.2 24.0  3.6e-4   17.8 11.1 19.0  3.6e-4; ...
    25 40.4 25.7 34.5  6.7e-4   39   24.3 34.0  6.7e-4; ...
    20 95.1 56.5 19.0  4.5e-4   87   53.7 45.0  4.5e-4; ...
    16  197  111  4.7  6.4e-4  162  101   36.0  6.4e-4; ...%stra
    11  374  247  4.1  0.017   350  220   21.0  6.9e-3; ...
    10  420  286  3.90 0.05    407  257   16.0  7.5e-3; ... %Ci
    08  526  378  3.90 0.25    522  347   12.0  0.035; ...
    06  650  492  4.30 0.85    661  463    6.4  0.21;...
    04  795  633  4.70 2.2     828  608    4.9  0.66; ...
    03  876  715 5.10  4.7     923  694    4.9  1.2; ...    %ST
    02  969  805 5.40  9.3    1037  790    4.9  1.8; ...
    01 1064  904 5.60  13.0   1162  897    5.4  2.5; ...
    00 1167 1013 5.60  19.0   1301 1018    6.0  3.5];
```

Physical variables are defined for a level z (first column). Layer values are calculated as arithmetic mean between top and bottom of the layer.

Units:
z(km), ro: density(g m^{-3}), p(hPa), O3(g m^{-3}), U: humidity(g m^{-3})

Relationship z(altitude km) → p(pressure level hPa): interpolation of Table data for z<=8 km yields

$$p(z=0)/p(z) = 1.0112 + 0.0885\ z + 0.0149\ z^2 \quad \text{(TROP)}, \quad \text{(A1.1)}$$
$$p(z=0)/p(z) = 1.0158 + 0.0927\ z + 0.0182\ z^2 \quad \text{(MLW)}. \quad \text{(A1.2)}$$

Deviation from the quotient is about +0.01 for z=0, being generally lower for higher levels. A simple exponentially decreasing fit for pressure shows higher deviations.

Parameterization of relationship p(pressure level hPa) → z(altitude km) :

$\eta = p/po$, with po= 1000 hPa, yields

$$z = 15.574 - 23.063\ \eta + 7.632\ \eta^2 \quad \text{(TROP)}, \quad \text{(A1.3)}$$
$$z = 14.757 - 22.330\ \eta + 7.7401\ \eta^2 \quad \text{(MLW)} \quad \text{(A1.4)}$$

Deviations are lower than 50 m if z<6 km.

ANNEX 2. Two-flux solutions for RTE

In order to describe upward E↑ and downward E↓ *diffuse irradiances*, the general RTE is reduced to a pair of first-order differential equations (τ' = optical depth; μo= cosZo; τ= total optical depth)

$$d\zeta\downarrow/d\tau' = -a_{11}\,\zeta\downarrow + a_{12}\,\zeta\uparrow + a_{13}\exp(-\tau'/\mu o), \qquad \text{(A2.1.a)}$$

$$d\zeta\uparrow/d\tau' = -a_{12}\,\zeta\downarrow + a_{22}\,\zeta\uparrow + a_{23}\exp(-\tau'/\mu o), \qquad \text{(A2.1.b)}$$

where irradiances are related to that incident on layer top: $\zeta\uparrow\downarrow = E\uparrow\downarrow(\tau')/E\downarrow(0)$. Coefficients are

$a_{11} = [1 - \omega\,(1-<b\downarrow>)]/<\mu\downarrow> - Edd$ $\qquad$ $a_{12} = \omega <b\uparrow>/<\mu\uparrow> - Edd$ $\qquad$ (A2.2.a)

$a_{21} = \omega <b\downarrow>/<\mu\downarrow> - Edd$ $\qquad$ $a_{22} = [1 - \omega\,(1-<b\uparrow>)]/<\mu\uparrow> - Edd$ $\qquad$ (A2.2.b)

$a_{13} = \omega\,(1 - bo)$ $\qquad$ $a_{23} = -\omega\,bo$ $\qquad$ (A2.2.c)

Exponential function in Eqs. (A2.1) describes attenuation of direct beam. Parameter bo is backscattered fraction for incident beam with zenithal angle Zo and describes probability of conversion to upward diffuse irradiance for scattered radiation. If only diffuse radiation propagates, then $a_{13} = a_{23} = 0$.

Edd=0 and g= 0 if pure isotropic radiation is assumed (Schuster-Schwartzschild approximation). Eddington (EDD) approximation admits axial symmetry around vertical direction (radiance is a function of τ' and μo) so that g>0 and Edd= ¼.

Brackets <X> refer to *hemispherical* mean value of a variable $X(\tau', \mathbf{\Omega})$ which is a function of all directions $\mathbf{\Omega}$. It is the case of diffuse radiance L, cosZ and backscattered fraction $b(\mathbf{\Omega}^i)$, $\mathbf{\Omega}^i$ = incident direction (Ceballos 1986 and 1988, Zdunkowski et al. 2007). For downward diffuse radiance,

$$<X\downarrow> = \int_{+2\pi} X(\mathbf{\Omega})\,L(\mathbf{\Omega})\,d\mathbf{\Omega} \,/ \int_{+2\pi} L(\mathbf{\Omega})\,d\mathbf{\Omega} \qquad \text{(A2.3)}$$

If diffuse radiance can be considered hemispherically isotropic, that is $L(\tau', \mathbf{\Omega}\downarrow) = L^{+}(\tau')$ and $L(\tau', \mathbf{\Omega}\uparrow) = L^{-}(\tau')$, then ([4])

$$<\mu\downarrow> = <\mu\uparrow> = \mu_m = ½, \qquad <b\downarrow> = <b\uparrow> = b_m, \qquad \alpha = b_m / \mu_m. \qquad \text{(A2.4)}$$

Parameters bo, b_m are defined by the scattering phase function $P(\mathbf{\Omega}', \mathbf{\Omega})$, with $\mathbf{\Omega}'$ being incident direction and $\mathbf{\Omega}$ scattered direction ([5]), with first-order approximations

$$bo(\mu o) = (1/4\pi) \int_{2\pi} P(\mathbf{\Omega}o, \mathbf{\Omega})\,d\Omega \approx ½(1 - 3\,g\,\mu o\,/2), \qquad \text{(A2.5)}$$

$$bm \approx ½(1 - 3\,g/4). \qquad \text{(A2.6)}$$

Integration is performed over backscattering hemisphere; eq. A2.6 assumes incidence of isotropic incident radiance.

Assuming hemispherical isotropy for diffuse radiance,

---

[4] EDD approximation operates considering hemispherical isotropy and the correction Edd= ¼. It could be suitable even for nearly isotropic Rayleigh scattering, in case of strong absorption (for instance, for UV radiation in Hartley-Huggins band interval).

[5] Phase function is a probability density, so that $(1/4\pi)\ P(\mathbf{\Omega}', \mathbf{\Omega})\,d\Omega$ is the probability of a photon with incident direction $\mathbf{\Omega}'$ being scattered within an element of solid angle $d\Omega$ with direction $\mathbf{\Omega}$. Additional information about phase function: see Liou (2002, ch. 3), Zdunkowski et al. (2007, ch 2).

- $a_{11} = a_{22}$, $a_{21} = a_{12}$. Solutions of Eqs. (A2.1) are described in Ceballos (2021).
- For conservative transfer, $\omega=1$ and $a_{11} = a_{12} = a_{21} = a_{22} = \alpha$.
- In the case of Rayleigh scattering we have g = 0, bm = ½, $\alpha=1$.

About solution of 2-flux RTE for conservative transfer

Consider a homogeneous layer with τ= optical depth, direct beam impinging on the top with zenithal angle Zo (cosZo = μo), over ground with reflectance Rs. We are interested in fluxes across the top and the base of such a layer. Boundary conditions are

$\zeta\downarrow(0)= 0, \qquad \zeta\uparrow(\tau)= Rs\ [\ \exp(\tau/\mu o) + \zeta\downarrow(\tau)]$.

We find for $\zeta\uparrow(0) = Rp$ (layer reflectance):

$$Rp(Rs>0) = \left(1 - (1-Rs)\ \{1 - \alpha\ \tau + (\alpha\ \mu o - bo)\ [1 - \exp(-\tau/\mu o)]\}\right) / [1 + (1-Rs)\ \alpha\ \tau], \qquad (A2.5)$$

$$Rp(Rs=0) = \{\alpha\ \tau - (\alpha\ \mu o - bo)\ [1 - \exp(-\tau/\mu o)]\}\ /\ [1 + \alpha\ \tau],$$

For global irradiance G on the surface,

$$G = (1 - Rp) / (1 - Rs), \qquad (A2.6)$$

$$G(Rs>0) = \{1 + (\alpha\ \mu o - bo)\ [1 - \exp(-\tau/\mu o)]\}/\ [1 + (1-Rs)\ \alpha\ \tau],$$

$$Go = G(Rs=0) = \{1 + (\alpha\ \mu o - bo)\ [1 - \exp(-\tau/\mu o)]\}/\ [1 + \alpha\ \tau].$$

Considering that G is the contribution of original Go plus multiple reflections (through counter-reflectance R*) within the layer,

$$G= Go / (1 - Rs\ R^*), \qquad (A2.7a)$$

$$G/Go = [1 + \alpha\ \tau]/[1 + (1-Rs)\ \alpha\ \tau] \qquad \rightarrow \qquad R^* = \alpha\ \tau / (1 + \alpha\ \tau). \qquad (A2.7b)$$

ANNEX 3. Radiative Code: Daily Cycle of UVNIR irradiance

```
%model GL0.2 for UVNIR

%Program GL02uvnirbis: calculation of daily cycle GLuvnir at two
altitudes
%and presents graphical results

% --------- local data -------
anomesdia= 20230820;
latitude= -30; longitude= -65;
Rso= [0.1 0.1];
zoo= [0.1 3];  %km
% --------------------------

astro= fastrododia(anomesdia);
delta= astro(1); dt= astro(2); D2Do= astro(3);

Souvnir= 757; S= Souvnir* D2Do;
Omega= 2*pi/24;
  UTC= [1:.5:24]'; Nt= length(UTC);
  G= zeros(Nt+2,2);

for altit= 1:2
  zo= zoo(altit); Rs= Rso(altit);
  po= 1018; Ap= 1.0158; Bp= 0.0927; Cp=  0.0182;  %MLW
  p= po/(Ap + Bp*zo + Cp*zo*zo);   %zo in km

  Kt= zeros(Nt,1); muo= zeros(Nt,1);
  Rp= Kt; As= Kt;
  for t=1:Nt
    tsolar= UTC(t)+(longitude/15) + dt;
    h= Omega*(12- tsolar);
    mu = cos(delta)*cosd(latitude)*cos(h) + sin(delta)*sind(latitude);
    if mu>0
      Ktbis= fKtuvnirbis(p, mu, Rs);
      Kt(t)= Ktbis(1); Rp(t)= Ktbis(2); As(t)= Ktbis(3);
      muo(t)= mu;
    endif
  endfor    % ciclo
  ha= acos(- tan(delta)*tand(latitude));
  UTC12= 12 - longitude/15 - dt;
  UTCrise= UTC12 - ha/Omega; UTCdown= UTC12+ ha/Omega;

  GLuvnir= S*muo.*Kt;
  GLuvnir= [GLuvnir; 0;0]; muo=[muo; 0;0];
    utc= [UTC; UTCrise; UTCdown];
    [utc, i]= sort(utc);
    G(:, altit)= GLuvnir(i);
    muo= muo(i);
    GLo= muo*S;

endfor  %altitudes
% ------------------------------------------------------------------

figure(1); clf
cyco= plot(utc, GLo, '-sm', 'MarkerSize', 3); hold on
cyc1= plot(utc, G(:,1), '-sc', 'MarkerSize', 3); hold on
cyc2= plot(utc, G(:,2), '-sk', 'MarkerSize', 3); hold on
plot(UTCrise ,0, 'sr'); hold on
plot(UTCdown, 0, 'sr'); hold on
```

```
title(['GLuvnir Daily Cycle - ' num2str(anomesdia)], 'FontSize', 13);
legend([cyco cyc1 cyc2], 'TOA', ['z ' num2str(zoo(1)) ' km'], ...
       ['z ' num2str(zoo(2))]);
xlabel('time UTC (hour)', 'FontSize', 12);
ylabel('irradiance (W m-2)', 'FontSize', 12);
to= 6;
latlon= ['[LAT, LON]= [' num2str([latitude longitude]) ']'];
text(to+1, 700, latlon); hold on
text(to+1, 660, ['altitude Z [' num2str(zoo) 'km ]']); hold on
text(to+1, 620, ['surface refl. Rs  [' num2str(Rso) ']']); hold on
axis([to 24 0 750])
grid
% --------------------------------------------------------------------
```

Function `Ktuvnirbis` calculates Kt index (interval UVNIR) for input `(p, μo, Rs)`

```
function Ktbis= fKtuvnirbis(p, muo, Rs)
%output [Ktuvnir Rp As] - Rp, As: components of Kto
% ----------------------------
psi= p/1000;
z= 14.757 - 22.330*psi + 7.7401 *psi*psi ;   %MLW
    % calculo Kto
    Aa= 0.342; Ba= 21.7; Ca= -3.28;
    As= Aa/(1 + Ba*muo + Ca*muo*muo);

    Ar= .353+ .099*psi;
    beta= 5.369; alfa= -0.860;  Br= beta*psi^alfa;
    Cr= 1.309*psi^3 - 3.530* psi^2 + 3.216*psi - 0.915;
    Rp= Ar/(1+ Br*muo + Cr*muo*muo);

    Kto= 1 - Rp - As;
    % ----------------------------
    % calculo Kt

    Bmed= -.334; Cmed= .171; Az= 0.102 - 0.008*z;  %z em km
    Ra= Az/(1+ Bmed*muo + Cmed*muo*muo);

    Ktuvnir= Kto/(1 - Rs * Ra);
    Ktbis= [Ktuvnir Rp As];
% ----------------------------
endfunction
```

Function `astrododia` evaluates declination δ, sun-earth relative distance $(D/Do)^2$ and time equation δt for input `anomesdia` .

```
 function astro= fastrododia(anomesdia)
    %versao corrigida abril 2017
    % calcula declinacao (radianos)e equacao do tempo (horas)
    %a partir da data anomesdia [yyyymmdd]
    dia0= [0 31 59 90 120 151 181 212 243 273 304 334];
    diasdoano= 365;
    ano= floor(anomesdia/10000); mesdia= mod(anomesdia,10000);
    mes= floor(mesdia/100); dia= mod(mesdia,100);
    if mod(ano,4)==0
        dia0= dia0 + [0 0 1 1 1 1 1 1 1 1 1 1];
        diasdoano= diasdoano+1;
    end
    diajul= dia0(mes)+ dia;
    teta = 2 * pi * (diajul-1) / diasdoano;  %radianos
    delta = .006918 - .399912 * cos(teta) + .070257 * sin(teta) -  ...
       .006758 * cos(2 * teta) + .000907 * sin(2 * teta) -  ...
       .002697 * cos(3 * teta) + .00148 * sin(3 * teta); %radianos
    dtempo= .000075 + .001868* cos(teta) - .032077* sin(teta) - ...
        .014615 * cos(2 * teta) - .040849* sin(2*teta); %radianos
    dt= dtempo*180/(pi*15); %horas

    r2sun = 1.00011 + .034221 * cos(teta) + .000128 * sin(teta) +...
        0.000719* cos(2*teta) + 0.000077* sin(2*teta);
    %algoritmo corrigido - Paltridge & Platt 1976
    %r2sun= (Do/D)^2
    astro= [delta; dt; r2sun];
endfuction
```